**A Theory of Decoy Value**

General Problem:

An enemy has M missiles that he wishes to use to destroy R assets in a missile attack.

We start with the following preliminary values. We have a number of Real Assets, R, a number of Decoy Assets D, and number of Missiles, M.

R = Number of Real Assets

D = Number of Decoys

M = Number of Missiles

We also model the probability that a given missile destroys a Real Asset with the following equation

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Assumption 1: Missiles always hit their identified target so:

= 1

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Assumption 2: Decoys are so good to be indistinguishable from real assets from the enemy point of view. So the probability of identifying the real assets for targeting is proportional to the possible targets.

and

Assumption 3: We’ll assume that missiles are dear so there is incentive to not waste missiles. For the first case, we assume that M = R. Thus:

and

The left-side equation states that the probability of kill any random Real asset R as a function of the number of decoys has an inverse relationship to the number of decoys while the right-side shows the probability of killing a decoy. The left-side equation is the one of interest.

Taking the partial derivative yields the marginal improvement in for each additional decoy added to the inventory.

We can express the expected value of having a fleet of decoys as a function of the number of decoys:

which simplifies to:

The marginal value of each drone can then be expressed by taking the partial derivative.

Given a or cost of each decoy, you’d theoretically maximize value of the decoy fleet by continuing to purchase decoys until

Maximize Overall Value where

Can we generalize for the other variables simultaneously?

So:

This equation allows for evaluation of changing Value at different combinations of R, , and D and allows for interesting analysis for different weapon systems of various quantities and costs. Particularly, for any R and , you could set equal to and solve for the number of drones you should theoretically purchase to protect your fleet of R assets.

With a preliminary framework in place, its easier to conceptualize a relaxation of the M=R assumption.

Extension to R+D > M > R:

The Value of the decoy fleet as a function of the number of Decoys is:

The Change in Value with respect to changes in D, R, R, , M:

To fill in the above equation, we find partial derivatives for all variables:

By Substitution:

We now have an equation that shows us the magnitude and direction of change of value, given any changes of the input variables D, R, , and M.

We should be able to incorporate decoy cost into the value equation for total value.

Total Value Equation:

Total Differential Equation:

Partial Derivatives

By Substitution:

So given M, R, , , you could solve for the ideal number of D by setting . The value of these equations is that they are generalizable beyond an Air Force context of very expensive aircraft vs. very cheap (in comparison) drones.